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Current algebra of super WZNW models*

E Abdalla†‡, M C B Abdalla‡§, O H G Branco‡ and L E Saltini‡

† CERN, Theory Division, 1211 Geneva 23, Switzerland

‡ Instituto de Física, Universidade de São Paulo, Cx. Postal 20516, BR-01498-970 São Paulo SP, Brazil

§ Instituto de Física Teórica, UNESP, Rua Pamplona 145, BR-01405, São Paulo SP, Brazil

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Abstract. We derive the current algebra of supersymmetric principal chiral models with a Wess–Zumino term. At the critical point one obtains two commuting super-affine Lie algebras as expected, but, in general, there are intertwining fields connecting both right and left sectors, analogously to the bosonic case. Moreover, in the present supersymmetric extension we have a quadratic algebra, rather than an affine Lie algebra, due to the mixing between bosonic and fermionic fields; the purely fermionic sector displays an affine Lie algebra as well.

Since the discovery of higher conservation laws for integrable models, algebraic methods have been frequently advocated in order to display the structure of the dynamics of field theory. Higher conserved charges imply non-trivial constraints for correlation functions, and one often finds such a strong algebraic machinery that a calculable S-matrix turns out to be a consequence. In the case of conformally invariant field theory, the Virasoro (and Kac–Moody) constraints fix all correlation functions.

Current algebra for integrable nonlinear sigma models were, up to very recently [1] largely unknown. Nevertheless, the Yang–Baxter relations are the best tools for exploring non-perturbative properties of integrable models [2]. The starting point of the formulation is, very generally, the consideration of the Poisson brackets between the spatial part of the Lax pair, leading to a Lie–Poisson algebra containing an antisymmetric numerical r-matrix which obeys the Yang–Baxter relation. The Yang–Baxter relation leads to an (almost) unique S-matrix of the theory [5–7, 17]. On the other hand, an algebraic strategy has also been effectively and successfully used in the case of 2D conformally invariant theories, in order to compute correlation functions [8], as well as in the case of two-dimensional gravity in the light-cone gauge [9].

Recently, the nonlinear σ-model with a Wess–Zumino term has been studied in this context [10]. The current, which corresponds to a piece of the Lax pair of the model [11], is shown to fulfill a new affine algebra. Such algebras have played an important role in several cases in field theory (see [12] and [13]). The hope is that after quantization we could be able to address the problem of computing exact Green functions, by means of higher conserved currents. Moreover, the results have been used in order to obtain the algebra of non-local charges [3]. As it turns out, the algebra is an extension of the affine Lie algebra, including cubic terms. Similar structures have appeared in the literature [4]. We expect to

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obtain similar structure in the supersymmetric case, where the S-matrix is known to be an extension of the purely bosonic counterpart, if not even simpler. With this aim, we study the current algebra in the supersymmetric sigma models.

In the present paper we consider the supersymmetric Wess–Zumino-Witten model [14, 15], which is defined by the action

$$S_{\rm susy}(g,\psi)=\frac{1}{\lambda^2}\int \mathrm{d}^2x\ \mathrm{tr}\,\partial^\mu g^{-1}\partial_\mu g+\frac{n}{4\pi}\int_0^1 \mathrm{d}r\int \mathrm{d}^2x\,\epsilon^{\mu\nu}\,\mathrm{tr}\,g_r^{-1}\partial_r g\,g_r^{-1}\partial_\mu g_r\,g_r^{-1}\partial_\nu g_r$$
$$+\frac{1}{4\lambda^2}\,\mathrm{tr}\int \mathrm{d}^2x\left\{\bar\psi\mathrm{i}\partial\!\!\!/\psi-\frac{1}{4}\left(1-\left(\frac{n\lambda^2}{4\pi}\right)^2\right)(\bar\psi\psi)^2+\mathrm{i}\frac{n\lambda^2}{4\pi}g^{-1}\partial^\mu g\,\bar\psi\gamma_5\gamma_\mu\psi\right\} \tag{1}$$

obtained from the superfield $G(x,\theta)=g(x)+\bar\theta\psi(x)+\frac{1}{2}\bar\theta\theta F(x)$, satisfying the constraint $G^\dagger(x,\theta)G(x,\theta)=1$, after integrating over the Grassman variable θ (see [14] for details, but noticing change in conventions see below and [17]).

The second term in the right-hand side of (1) is the so-called Wess–Zumino term and only depends linearly on the time derivative of the field g. Hence it proves useful to rewrite it in terms of $A(g)$, introduced in order to permit the canonical quantization of the theory [16, 17], being defined in such a way that

$$\int_0^1 \mathrm{d}r\int \mathrm{d}^2x\varepsilon^{\mu\nu}\ \mathrm{tr}\,g_r^{-1}\partial_r g_r\,g_r^{-1}\partial_\mu g_r\,g_r^{-1}\partial_\nu g_r\equiv\int \mathrm{d}^2x\ \mathrm{tr}\ A(g)\partial_0 g \tag{2}$$

and

$$\frac{\partial A_{ij}}{\partial g_{lk}}-\frac{\partial A_{kl}}{\partial g_{ji}}=\partial_1 g_{il}^{-1}g_{kj}^{-1}-g_{il}^{-1}\partial_1 g_{kj}^{-1}\,. \tag{3}$$

We shall use the following notation: $\alpha=n\lambda^2/4\pi$, $\varepsilon^{01}=1$, $\gamma^0=\left(\begin{smallmatrix}0&-\mathrm{i}\\ \mathrm{i}&0\end{smallmatrix}\right)$, $\gamma^1=\left(\begin{smallmatrix}0&\mathrm{i}\\ \mathrm{i}&0\end{smallmatrix}\right)$, and $\gamma^5=\gamma^0\gamma^1$. When $\alpha=\pm1$ we have a super conformally invariant theory. Canonical quantization is straightforward, on account of [10, 16, 17]; we have the following canonically conjugated momenta:

$$\bar\Pi^g_{ij}=\frac{\partial\mathcal{L}}{\partial(\partial_0 g_{ij})}=\frac{1}{\lambda^2}\partial^0 g_{ji}^{-1}-\frac{n}{4\pi}A_{ji}(g)+\frac{\mathrm{i}\alpha}{4\lambda^2}\bar\psi_{jk}\gamma^1\psi_{km}g_{mi}^{-1} \tag{4}$$

and

$$\Pi^\psi_{ij}=\frac{\partial\mathcal{L}}{\partial(\partial_0\psi_{ij})}=\frac{\mathrm{i}}{4\lambda^2}\bar\psi_{ji}\gamma_0=\frac{\mathrm{i}}{4\lambda^2}\psi^\dagger{}_{ji} \tag{5}$$

where the first term in the right-hand side of (4) is the momentum canonically conjugated to the field g, in the principal chiral model only

$$\Pi^g_{ij}=\frac{1}{\lambda^2}\partial^0 g_{ji}^{-1} \tag{6}$$

with the following Poisson algebra:

$$\begin{aligned}
&\{g_{ij}(x),g_{kl}(y)\}=0 && \{\bar\Pi^g_{ij}(x),\bar\Pi^g_{kl}(y)\}=0\\
&\{g_{ij}(x),\bar\Pi^g_{kl}(y)\}=\delta_{ik}\delta_{jl}\delta(x_1-y_1) && \{\psi_{ij}(x),\psi_{kl}(y)\}_+=0\\
&\{\Pi^\psi_{ij}(x),\Pi^\psi_{kl}(y)\}_+=0 && \{\psi_{ij}(x),\Pi^\psi_{kl}(y)\}_+=\delta_{ik}\delta_{jl}\delta(x_1-y_1)\,.
\end{aligned} \tag{7}$$

However, as we have already mentioned this model has been obtained via a superfield formulation where we have imposed the constraint

$$G^\dagger(x,\theta)G(x,\theta)=1 \tag{8}$$

which leads, for the fermion component, to the relation

$$\psi^{\dagger}{}_{ij} = -g^{-1}_{ik}\psi_{kl}g^{-1}_{lj} \,. \tag{9}$$

In the phase space we have the constraint

$$\Omega_{il} = \Pi^{\psi}_{li} + \frac{\mathrm{i}}{4\lambda^2} g^{-1}_{im}\psi_{mk}g^{-1}_{kl} \tag{10}$$

which must be implemented using the Dirac method [18]. The basic element of the method is the dirac matrix

$$Q = \{\Omega_{ij}(x), \Omega_{kl}(y)\} = \frac{\mathrm{i}}{2\lambda^2} g^{\dagger}{}_{il} g^{\dagger}{}_{kj}\delta(x_1 - y_1) \tag{11}$$

satisfying

$$\int \mathrm{d}z \{\Omega_{ij}(x), \Omega_{mn}(z)\}\{\Omega_{mn}(z), \Omega_{kl}(y)\}^{-1} = \delta_{ik}\delta_{jl}\delta(x_1 - y_1) \,. \tag{12}$$

From Q we can obtain the Dirac brackets (notice that as we have $\{g_{ij}(x), \Omega_{kl}(y)\} = 0$ the Dirac bracket of g_{ij} with any other field is the Poisson bracket itself). Writing (4) for the field Π^g_{ij} instead of the field $\bar{\Pi}^g_{ij}$, they read

$$\begin{aligned}
&\left\{g_{ij}(x), g_{kl}(y)\right\}_D = 0 \qquad \{g_{ij}(x), \psi_{kl}(y)\}_D = 0 \\
&\left\{g_{ij}(x), \Pi^{\psi}_{kl}(y)\right\}_D = 0 \qquad \{g_{ij}(x), \Pi^{g}_{kl}(y)\}_D = \delta_{ik}\delta_{jl}\delta(x_1 - y_1) \\
&\left\{\Pi^g_{ij}(x), \Pi^g_{kl}(y)\right\}_D = \left\{\tfrac{1}{2}(1-\alpha^2)\left[g^{\dagger}_{jk}\Pi^{\psi}_{ml}\psi_{mn}g^{\dagger}_{ni} - g^{\dagger}_{li}\Pi^{\psi}_{mj}\psi_{mn}g^{\dagger}_{nk}\right] + \frac{\alpha}{\lambda^2}F_{ji,lk}\right\}\delta(x_1 - y_1) \\
&\left\{\Pi^g_{ij}(x), \psi_{kl}(y)\right\}_D = -\tfrac{1}{2}\left\{\delta_{ik}\left[g^{\dagger}_{jm}\psi_{ml} - \alpha g^{\dagger}_{jm}\gamma_5\psi_{ml}\right] + \delta_{jl}\left[\psi_{km}g^{\dagger}_{mi} + \alpha\psi_{km}\gamma_5 g^{\dagger}_{mi}\right]\right\}\delta(x_1 - y_1) \\
&\left\{\Pi^g_{ij}(x), \Pi^{\psi}_{kl}(y)\right\}_D = \tfrac{1}{2}\left\{g^{\dagger}_{li}\Pi^{\psi}_{kj} + \Pi^{\psi}_{il}g^{\dagger}_{jk} - \alpha\left[\Pi^{\psi}_{il}\gamma_5 g^{\dagger}_{jk} - g^{\dagger}_{li}\gamma_5\Pi^{\psi}_{kj}\right]\right\}\delta(x_1 - y_1) \qquad (13) \\
&\{\psi_{ij}(x), \psi_{kl}(y)\}_D = -\frac{2\lambda^2}{\mathrm{i}} g_{il}g_{kj}\delta(x_1 - y_1) \\
&\left\{\psi_{ij}(x), \Pi^{\psi}_{kl}(y)\right\}_D = \tfrac{1}{2}\delta_{ik}\delta_{jl}\delta(x_1 - y_1) \\
&\left\{\Pi^{\psi}_{ij}(x), \Pi^{\psi}_{kl}(y)\right\}_D = \frac{-\mathrm{i}}{8\lambda^2} g^{\dagger}_{li}g^{\dagger}_{jk}\delta(x_1 - y_1)
\end{aligned}$$

where

$$F_{ij,kl} = (g^{\dagger}_{il})'(x)g^{\dagger}_{kj}(x) - g^{\dagger}_{il}(x)(g^{\dagger}_{kj})'(x) \,.$$

We are now ready to study the relevant algebraic properties of the supersymmetric WZW theory. In analogy with the bosonic case [10], we consider the conserved Noether currents, which in the supersymmetric case are given by

$$J^{\mathrm{R}}_{\pm,a}(x) = -\frac{1}{\lambda^2}\,\mathrm{tr}\left\{(1 \mp \alpha)\left[g((\dot{g}^{\dagger}) \pm (g^{\dagger})') + \tfrac{1}{4}\mathrm{i}\psi(1 \pm \gamma_5)\psi^{\dagger}\right]t_a\right\}(x) \tag{14}$$

and

$$J^{\mathrm{L}}_{\pm,a}(x) = -\frac{1}{\lambda^2}\mathrm{tr}\left\{(1 \pm \alpha)\left[(-(\dot{g}^{\dagger}) \mp (g^{\dagger})')g + \tfrac{1}{4}\mathrm{i}\psi^{\dagger}(1 \pm \gamma_5)\psi\right]t_a\right\}(x) \,. \tag{15}$$

Here we introduced the notation with the indices of a basis t_a of the Lie algebra G where the fields g are defined, with the structure constants f^c_{ab} defined as

$$[t_a, t_b] = f^c_{ab}t_c$$

and any field $J^{\mathrm{R\,or\,L}}_{\pm,a}$ is defined as

$$J^{\mathrm{R\,or\,L}}_{\pm,a} = \left(J_{\pm}, t^{\mathrm{R\,or\,L}}_a\right) = -\,\mathrm{tr}\left(J^{\mathrm{R\,or\,L}}_{\pm}t_a\right).$$

In addition, it is important in the case of theories containing fermions to introduce the fermionic currents,

$$i^{\mathrm{R}}_{\pm,a}(x) = -\frac{\mathrm{i}}{4\lambda^4}\ \mathrm{tr}[\psi(1\mp\gamma_5)\psi^\dagger t_a](x) \tag{16}$$

and

$$i^{\mathrm{L}}_{\pm,a}(x) = -\frac{\mathrm{i}}{4\lambda^4}\ \mathrm{tr}[\psi^\dagger(1\mp\gamma_5)\psi t_a](x)\,. \tag{17}$$

Since we know, from the case of the study of supersymmetric non-local charges, that the above purely fermionic objects do also show up independently [17, 19, 20] of the currents [12]. Moreover, there are also intertwining operators already in the bosonic case [10, 1], which are described by the fields

$$j_{ab}(x) = \frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a g t_b](x)\,. \tag{18}$$

In the case of supersymmetric theories we also have the fermionic partner

$$j^{\psi}{}^{\mathrm{R}}_{\pm,ab}(x) = \frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger\ i^{\mathrm{R}}_{\pm} t_a g t_b](x) \tag{19}$$

and

$$j^{\psi}{}^{\mathrm{L}}_{\pm,ab}(x) = \frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a\, g i^{\mathrm{L}}_{\pm}\ t_b](x)\,. \tag{20}$$

We shall see that after coupling bosons with fermions, an infinite number of fields will be present, as a consequence of a quadratic algebra, which is absent in the purely bosonic, as well as in the purely fermionic models. We introduce

$$K^{\mathrm{R}}_{\pm,abc}(x) = -\frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a i^{\mathrm{R}}_{\pm} t_b g t_c](x) \tag{21}$$

$$K^{\mathrm{L}}_{\pm,abc}(x) = -\frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a g t_b i^{\mathrm{L}}_{\pm} t_c](x) \tag{22}$$

$$Y^{\mathrm{R}}_{\pm,abcd}(x) = \frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a g t_b g^\dagger i^{\mathrm{R}}_{\pm} t_c g t_d](x) \tag{23}$$

$$Y^{\mathrm{L}}_{\pm,abcd}(x) = \frac{1}{\lambda^2}\ \mathrm{tr}[g^\dagger t_a g t_b g^\dagger t_c g i^{\mathrm{L}}_{\pm} t_d](x)\,. \tag{24}$$

We are now in position to write down the full algebra. The purely right sector is very simple. For the purely left sector one substitutes ($\mathrm{R}\to\mathrm{L}$) and $\alpha\to-\alpha$. We have

$$\begin{aligned}\{J^{\mathrm{R}}_{\pm,a}(x), J^{\mathrm{R}}_{\pm,b}(y)\} = &-\tfrac{1}{2}(1\mp\alpha) f^c_{ab}\{(3\pm\alpha)J^{\mathrm{R}}_{\pm,c} - (1\mp\alpha)J^{\mathrm{R}}_{\mp,c}\\ &+\frac{\lambda^2}{2}(1\mp\alpha)[(3\pm 2\alpha-\alpha^2)i^{\mathrm{R}}_{\pm,c} - (5\mp 2\alpha+\alpha^2)i^{\mathrm{R}}_{\mp,c}]\}\delta(x_1-y_1)\\ &\pm 2(1\mp\alpha)^2\eta_{ab}\delta'(x_1-y_1)\end{aligned} \tag{25}$$

$$\begin{aligned}\{J^{\mathrm{R}}_{\pm,a}(x), J^{\mathrm{R}}_{\mp,b}(y)\} = &-\tfrac{1}{2} f^c_{ab}\{(1-\alpha)^2 J^{\mathrm{R}}_{-,c} + (1+\alpha)^2 J^{\mathrm{R}}_{+,c}\\ &-(1-\alpha^2)^2\frac{\lambda^2}{2}[i^{\mathrm{R}}_{-,c} + i^{\mathrm{R}}_{+,c}]\}\delta(x_1-y_1)\end{aligned} \tag{26}$$

$$\{i^{\mathrm{R}}_{\pm,a}(x), J^{\mathrm{R}}_{\pm,b}(y)\} = -\tfrac{1}{2}(1\mp\alpha)^2 f^c_{ab} i^{\mathrm{R}}_{\pm,c}\delta(x_1-y_1) \tag{27}$$

$$\{i^{\mathrm{R}}_{\pm,a}(x), J^{\mathrm{R}}_{\mp,b}(y)\} = \tfrac{1}{2}(1\pm\alpha)^2 f^c_{ab} i^{\mathrm{R}}_{\pm,c}\delta(x_1-y_1) \tag{28}$$

$$\{i^{\mathrm{R}}_{\pm,a}(x), i^{\mathrm{R}}_{\pm,b}(y)\} = \frac{1}{\lambda^2} f^c_{ab} i^{\mathrm{R}}_{\pm,c}\delta(x_1-y_1) \tag{29}$$

$$\{i^{R}_{\pm,a}(x), i^{R}_{\mp,b}(y)\} = 0 \tag{30}$$

where

$$\eta_{ab} = (t_a, t_b) = -\frac{1}{\lambda^2}\,\mathrm{tr}(t_a t_b)\,.$$

Notice that the intertwining operators did not appear at all up to this point, the mixed sector is more involved. We have

$$\begin{aligned}\left\{J^{R}_{\pm,a}(x), J^{L}_{\pm,b}(y)\right\} &= \pm(1-\alpha^2)\left[(1\pm\alpha)j_{ab}(x) + (1\mp\alpha)j_{ab}(y)\right]\delta'(x_1-y_1)\\ &\quad -(1-\alpha^2)\frac{\lambda^2}{4}\Big[(1-\alpha^2)\Big(j^{\psi R}_{-ab} + j^{\psi R}_{+ab} - j^{\psi L}_{-ab} - j^{\psi L}_{+ab}\Big)\\ &\quad +8\Big(j^{\psi R}_{\mp ab} - j^{\psi L}_{\mp ab}\Big)\Big]\delta(x_1-y_1)\end{aligned} \tag{31}$$

$$\begin{aligned}\left\{J^{R}_{\pm,a}(x), J^{L}_{\mp,b}(y)\right\} &= (1-\alpha^2)\Big[\mp(1\mp\alpha)j'_{ab}(x)\\ &\quad -\frac{\lambda^2}{4}(1\mp\alpha)(3\mp\alpha)\Big[j^{\psi R}_{-ab} + j^{\psi R}_{+ab} - j^{\psi L}_{-ab} - j^{\psi L}_{+ab}\Big]\Big]\delta(x_1-y_1)\end{aligned} \tag{32}$$

$$\left\{i^{R}_{\pm,a}(x), J^{L}_{\pm,b}(y)\right\} = \tfrac{1}{2}(1-\alpha^2)\Big[j^{\psi L}_{\pm ab} - j^{\psi R}_{\pm ab}\Big]\delta(x_1-y_1) \tag{33}$$

$$\left\{i^{R}_{\pm,a}(x), J^{L}_{\mp,b}(y)\right\} = \tfrac{1}{2}(1\mp\alpha)(3\mp\alpha)\Big[j^{\psi L}_{\pm ab} - j^{\psi R}_{\pm ab}\Big]\delta(x_1-y_1) \tag{34}$$

$$\left\{i^{L}_{\pm,a}(x), J^{R}_{\pm,b}(y)\right\} = -\tfrac{1}{2}(1-\alpha^2)\Big[j^{\psi L}_{\pm ba} - j^{\psi R}_{\pm ba}\Big]\delta(x_1-y_1) \tag{35}$$

$$\left\{i^{L}_{\pm,a}(x), J^{R}_{\mp,b}(y)\right\} = -\tfrac{1}{2}(1\pm\alpha)(3\pm\alpha)\Big[j^{\psi L}_{\pm ba} - j^{\psi R}_{\pm ba}\Big]\delta(x_1-y_1) \tag{36}$$

$$\left\{i^{R}_{\pm,a}(x), i^{L}_{\pm,b}(y)\right\} = \frac{1}{\lambda^2}\Big[j^{\psi L}_{\pm ab} - j^{\psi R}_{\pm ab}\Big]\delta(x_1-y_1) \tag{37}$$

$$\left\{i^{R}_{\pm,a}(x), i^{L}_{\mp,b}(y)\right\} = 0\,. \tag{38}$$

We see now in (31) and (32) the explicit appearance of the bosonic intertwiners as well as the fermionic one in (31)–(37). Notice also the simplicity of the purely fermionic components brackets. In the critical case, $\alpha \to +1$ we see that a whole sector decouples completely, the same being true for $\alpha \to -1$. We have in that case a super-affine algebra, as expected, and the model is completely soluble. In fact, the conserved charges can be used in order to provide a complete solution of the Green functions, the only missing ingredient with respect to our computation being the super-Virasoro generators. We finally write down the part of the algebra involving the intertwiners. We have

$$\left\{J^{L}_{\pm,a}(x), j_{bc}(y)\right\} = -(1\pm\alpha)f^{d}_{ac}j_{bd}\delta(x_1-y_1) \tag{39}$$

$$\left\{J^{R}_{\pm,a}(x), j_{bc}(y)\right\} = -(1\mp\alpha)f^{d}_{ab}j_{dc}\delta(x_1-y_1) \tag{40}$$

$$\left\{j_{ab}(x), j_{cd}(y)\right\} = 0 \tag{41}$$

$$\left\{j_{ab}(x), i^{L}_{\pm,c}(y)\right\} = 0 \tag{42}$$

$$\left\{j_{ab}(x), i^{R}_{\pm,c}(y)\right\} = 0 \tag{43}$$

$$\left\{j_{ab}(x), j^{\psi R}_{\pm,cd}(y)\right\} = 0 \tag{44}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, J^{L}_{\pm,c}(y)\right\} = -\tfrac{1}{2}(1\pm\alpha)\Big[2j^{\psi R}_{\pm\overline{ac},b} - (1\pm\alpha)j^{\psi R}_{\pm\overline{ab},c} - (1\mp\alpha)K^{L}_{\pm,abc}\Big]\delta(x_1-y_1) \tag{45}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, J^{L}_{\mp,c}(y)\right\} = -\tfrac{1}{2}(1\mp\alpha)\Big[2j^{\psi R}_{\pm\overline{ac},b} + (1\mp\alpha)j^{\psi R}_{\pm\overline{ab},c} - (3\mp\alpha)K^{L}_{\pm,abc}\Big]\delta(x_1-y_1) \tag{46}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, J^{R}_{\pm,c}(y)\right\} = \tfrac{1}{2}(1\mp\alpha)\Big[2j^{\psi R}_{\pm\underline{ac},b} - (1\mp\alpha)j^{\psi R}_{\pm\underline{ca},b} - (1\pm\alpha)K^{R}_{\pm cab}\Big]\delta(x_1-y_1) \tag{47}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, J^{R}_{\mp,c}(y)\right\} = \tfrac{1}{2}(1\pm\alpha)\Big[2j^{\psi R}_{\pm\underline{ac},b} + (1\pm\alpha)j^{\psi R}_{\pm\underline{ca},b} - (3\pm\alpha)K^{R}_{\pm cab}\Big]\delta(x_1-y_1) \tag{48}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, i^{L}_{\pm,c}(y)\right\} = -\frac{1}{\lambda^2}\Big[j^{\psi R}_{\pm\overline{ab},c} - K^{L}_{\pm abc}\Big]\delta(x_1-y_1) \tag{49}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, i^{L}_{\mp,c}(y)\right\} = 0 \tag{50}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, i^{R}_{\pm,c}(y)\right\} = \frac{1}{\lambda^2}\Big[j^{\psi R}_{\pm\underline{ca},b} - K^{R}_{\pm cab}\Big]\delta(x_1-y_1) \tag{51}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, i^{R}_{\mp,c}(y)\right\} = 0 \tag{52}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, j^{\psi L}_{\pm,cd}(y)\right\} = \frac{1}{\lambda^2}\big[K^{L}_{\pm\underline{cab},d} - K^{R}_{\pm\overline{cab},d}\big]\delta(x_1-y_1) \tag{53}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, j^{\psi L}_{\mp,cd}(y)\right\} = 0 \tag{54}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, j^{\psi R}_{\pm,cd}(y)\right\} = \frac{1}{\lambda^2}\big[Y^{R}_{\pm abcd} - Y^{R}_{\pm cdab}\big]\delta(x_1-y_1) \tag{55}$$

$$\left\{j^{\psi R}_{\pm,ab}(x),\, j^{\psi R}_{\mp,cd}(y)\right\} = 0 \tag{56}$$

where

$$\begin{aligned}
\left(j^{\psi R}_{\pm\overline{ab}}\right)_{ij} &= \frac{1}{\lambda^2}\left(g^\dagger i^R_\pm t_a g t_b\right)_{ij} & \left(j^{\psi R}_{\pm\underline{ab}}\right)_{ij} &= \frac{1}{\lambda^2}\left(g^\dagger i^R_\pm t_a t_b g\right)_{ij}\\
\left(K^R_{\pm\overline{abc}}\right)_{ij} &= -\frac{1}{\lambda^2}\left(g^\dagger t_a i^R_\pm t_b g t_c\right)_{ij} & \left(K^R_{\pm\underline{abc}}\right)_{ij} &= -\frac{1}{\lambda^2}\left(g^\dagger t_a t_b i^R_\pm t_c g\right)_{ij}\\
\left(j^{\psi L}_{\pm\overline{ab}}\right)_{ij} &= \frac{1}{\lambda^2}\left(g^\dagger t_a g i^L_\pm t_b\right)_{ij} & \left(j^{\psi L}_{\pm\underline{ab}}\right)_{ij} &= \frac{1}{\lambda^2}\left(g^\dagger t_a t_b g i^L_\pm\right)_{ij}\\
\left(K^L_{\pm\overline{abc}}\right)_{ij} &= -\frac{1}{\lambda^2}\left(g^\dagger t_a g t_b i^L_\pm t_c\right)_{ij} & \left(K^L_{\pm\underline{abc}}\right)_{ij} &= -\frac{1}{\lambda^2}\left(g^\dagger t_a t_b g t_c i^L_\pm\right)_{ij}\,.
\end{aligned}$$

It is clear now that the algebra is quadratic. Therefore, a new structure arises in the case of supersymmetric theories involving the WZW term. Indeed, integrability of the bosonic model [11, 21] does not arise in the same manner in the supersymetric case [14]. However, it is certainly true that for $\alpha = 0$ the theory must be integrable. Indeed, there is a considerable simplification, but the algebra obtained is still quadratic. If the algebra of non-local charges should follow the pattern of the purely bosonic case, the quadratic algebra thus obtained must be severely constrained.

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